The Varying Risk Market Model: A Reexamination Based On Heteroskedastic Conditions and Other Statistical Robustness Tests

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Abstract

Bhardwaj and Brooks (1993) estimates of a varying risk market model indicate that betas of market value ranked stock portfolios are larger in bull market months for small market value portfolios and smaller for large market value portfolios. This paper investigates the statistical robustness of these and other Bhardwaj and Brooks findings by examining multicollinearity, autocorrelation, and heteroskedasticity diagnostics for the constant and varying risk models and thus the models' validity. Regressions and diagnostics for two equal sub-periods of the Bhardwaj and Brooks regressions are also generated as a further robustness check. Evidence is provided that there is a significant difference in heteroskedasticity between bull and bear markets for the varying risk market model. Also, a GARCH(1,1) model is proposed, after estimation, to account for the time-varying heteroskedasticity identified. Based on the diagnostics conducted, the varying risk market model is statistically robust.

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INTRODUCTION

The Sharpe-Lintner-Black Capital Asset Pricing Model (CAPM) proposes that security returns are a positive linear function of their market betas and that these betas are sufficient to describe the cross-section of expected security returns. A number of studies support the basic premise of the CAPM but recent examinations such as those by Fama and French (1992) cast doubt on the central prediction of the CAPM that stock returns are positively and linearly related to market betas. Instead, they find that firm size and book-to-market value of equity explain security returns with more significance when substituted for market beta. Conrad and Kaul (1988) and Ferson and Harvey (1991) show that a substantial portion of the variance in realized returns is inversely related to firm size and that variations in expected returns change systematically over time.

Since the factors considered in these studies displace market betas in explaining the cross-section of expected security returns, several recent studies have reexamined the relevance of the CAPM in explaining security returns by investigating if risk premiums and thus security returns vary based on economic cycles. Fabozzi and Francis (1977) find neither risk nor abnormal returns are significantly affected by bull and bear markets. However, Fabozzi and Francis (1979) find that intercept and beta coefficients both tend to experience statistically significant shifts during periods of economic contraction and expansion. Fama and French (1989) also observe a business cycle pattern in expected returns on common stocks and bonds.

Bhardwaj and Brooks (1993) provide evidence that betas vary between bull and bear markets with significantly larger betas in bull market months for small market value portfolios but with significantly smaller betas for large market value portfolios in bull markets. Small market value portfolio return variance is found to be higher in bull markets with small market value return variance in recessions being the major reason for the greater volatility of market returns (CRSP equally–weighted index) in recessions. Bhardwaj and Brooks also document a correlation between economic cycles and bull/bear markets with bull markets coinciding with recessions. Thus, their results support a varying risk market model over a constant risk model.

The purpose of this study is first, to review the statistical robustness of Bhardwaj and Brooks' findings including examining their results over two equal sub-periods and, second, to extend their model by incorporating GARCH modeling to account for the presence of time-varying heteroskedasticity. Multicollinearity and autocorrelation diagnostics indicate the Bhardwaj and Brooks varying risk model is robust to these tests. However, White's test and the modified Breusch-Pagan test indicate the presence of heteroskedasticity in both the constant and varying risk models. Also, heteroskedasticity is found to vary significantly between bull and bear markets with the bull market residuals significantly larger. Time-varying heteroskedasticity is indicated by Engle Lagrange multiplier tests and by the AR(1)-GARCH(1,1) model estimated and then adopted to account for the time-varying heteroskedasticity. The Bhardwaj and Brooks varying risk model estimates are found to be, however, robust to time-varying heteroskedasticity and robust when subperiods of data are tested.

DATA AND METHODOLOGY

Twenty portfolios, ranked by total market value of equity, of NYSE (and AMEX, after 1962) stocks are assembled for each of the years between 1926-1990. MV1 is the smallest market value portfolio while

MV20 is the largest market value portfolio. The CRSP equally-weighted return index with dividends included (EWI) proxies for the market portfolio. The median value of EWI for the time period of this study is 1.3835%. Monthly index returns above (below) the median are classified as bull (bear). This provides equal subsets of 390 observations each for bull and bear markets between 1926 and 1990. The returns used are drawn from the CRSP monthly master file for 1926-1990. These include NYSE security returns over the entire period and AMEX returns since 1963. NASDAQ returns are not available for most of the period investigated and thus NASDAQ firms are not included. Thus, extremely small firms are not included in the portfolios. Ibbotson monthly holding period returns on T-bills proxy for the risk-free rate. The portfolios are assembled as follows:

1. All common equity (stock) securities with 12 monthly returns available for a given calendar year are initially retained. Securities with missing monthly returns or which are delisted or initially listed during a given year are not included. Securities for firms which are merged or acquired within the year are also not included.

2. Among the securities in (1) above, those with recorded shares outstanding and a stock price greater than 0 at the end of the previous calendar year are retained. Securities with a missing stock price or missing shares outstanding values or with either value equal to 0 are discarded.

3. The number of securities retained given (1) and (2) above are provided in Table 1. Each of the 20 market value ranked portfolios is assigned securities based on the market value of equity at the end of the previous calendar year. Each portfolio retains its security composition for one year with securities reassigned to other portfolios as values change from year to year.

4. Over the period examined (1926-1990), the number of securities retained each year is divided by 20 to find the minimum number of securities per portfolio. Any "left-over" securities not evenly divided by 20 (from 1 to 19) are randomly assigned at the beginning of the year to the 20 portfolios. For instance, with 801 securities, each portfolio initially will have 40 securities and there is one security "left-over". Assuming portfolio 2 is randomly drawn from the 20 portfolios, portfolio 2 then will have 41 securities and all other portfolio will have 40 securities. Next, the specific securities are assigned to the portfolios based on market value of equity with the first 40 in portfolio 1, the next 41 in portfolio 2 and 40 securities in the remaining portfolios 3 through 20.

5. Monthly portfolio returns are generated by an equally-weighted average of the monthly returns on the stocks in a given portfolio.

A summary of the range of portfolio market values and average market values is provided in Table 2.

{Insert Tables 1 and 2}

Table 2 also provides descriptive information on the return behavior of the 20 market value portfolios between 1926 and 1990. In general, returns and standard deviations of returns are inversely related to portfolio values. All 20 portfolios exhibit positive bull and negative bear month returns on average. Small firm stocks exhibit the greatest return variance and perform more poorly during bear markets. Small firm stocks also exhibit more variation in returns between bear and bull months than large firm stocks. The standard deviation of returns for portfolio MV1 in bull months is 15.83% while it is only 6.89% in bear months. The bull and bear market return standard deviations for portfolio MV20 are, however, only, respectively, 6.84% and 5.10%. The greater bull month standard deviation of returns is also consistent, assuming bull months occur more often during recessions, with Schwert (1989) who finds the stock market more volatile during recessions.

The *constant risk market model* for each market portfolio is:

(1) $R_t = a_1 + b_1 R_{mt} + e_{1t}$

The *varying risk market model* used in this study is:

(2) $R_t = a_2 + a_3 D_{1t} + b_2 R_{mt} + b_3 R_{mt} D_{1t} + e_{2t}$

or

$$R_t = a_{bull} + (a_{bear} - a_{bull}) D_{1t} + b_{bull} R_{mt} + (b_{bear} - b_{bull}) R_{mt} D_{1t} + e_{2t}$$

where,

R_t = return on a portfolio in excess of the risk-free rate in month t,

R_{mt} = return on the market proxy portfolio in excess of the risk-free rate in month t, and

D_{1t} = a dummy variable equal to 1 for bear months and 0 for bull months.

The intercept a_1 and slope b_1 from (1) represent the average abnormal return and systematic risk, respectively, over the entire period, for the constant risk market model. The parameter estimates a_2 and ($a_2 + a_3$), from (2), represent average abnormal returns in bull and bear markets, respectively, while b_2 and ($b_2 + b_3$), from (2), represent corresponding estimates of systematic risk for the varying risk market model. If a_{bear} is statistically different from a_{bull} (a_3 statistically significant), and b_{bear} is statistically different from b_{bull} (b_3 statistically significant), then abnormal returns and systematic risk, respectively, differ between bull and bear markets.[1] F-tests are also conducted to test the joint hypotheses, $a_{bear} = a_{bull}$ and $b_{bear} = b_{bull}$, to indicate if there is a statistical difference between the constant risk and varying risk market models.

Regression results using the constant and varying risk models of (1) and (2) are estimated for each of the twenty portfolio time-series over the period 1926-1990 with the results provided in Table 3. Overall results support prior studies of systematic risk variability over the business cycle.[2] According to the constant risk model, the systematic risks of the smallest and largest value portfolios are 1.59 and .621, respectively. Allowing market risk to vary between bull and bear markets indicates a higher beta of 1.98 in bull months compared to 1.15 in bear months for the smallest value portfolio, MV1. The market risk (or beta) of the largest-valued portfolio, MV20, however, is higher in bear months than in bull months with betas of .75 (bear) and .49 (bull). These preliminary results support the findings of Bhardwaj and Brooks. F-tests indicate a statistical difference between the constant risk and varying risk models for all twenty portfolios.

{Insert Table 3}

Financially-distressed firms tend to shed assets and decrease in total market value of equity and thus would tend to migrate to the small market value portfolios, especially in times of recessions. This would also explain the higher betas for the small market value portfolios and the larger difference between small

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[1]The values of a_{bear} and b_{bear} are found by summing the regression parameter estimates for a_2 and a_3 ($= a_{bull} + a_{bear} - a_{bull}$) and b_2 and b_3 ($= b_{bull} + b_{bear} - b_{bull}$), respectively.

[2]For instance, Fama and French (1989) indicate and provide evidence that when business conditions are poor (with lower income levels), expected returns for stocks (and bonds) are higher so that funds will be moved from consumption to investment.

market value portfolio betas and large market value portfolio betas in bull markets, if as Bhardwaj and Brooks indicate, bull markets tend to occur during recessions when more financially distressed firms would be encountered.

The abnormal returns in Table 3 are smaller than those estimated by Bhardwaj and Brooks. The significance of the a_{bear}-a_{bull} parameter in the varying risk model regressions indicates that the abnormal returns are statistically different in bull and bear markets with the abnormal returns smaller in bull markets for the smaller value portfolios but larger for the larger value portfolios in bull markets. There is a surprising number of significant abnormal returns during bull markets.

MODEL EVALUATION

Ordinary least squares (OLS) estimation has distinct advantages. If certain statistical assumptions are met, termed the classical linear regression (CLR) model, OLS estimation is consistent and asymptotically efficient and estimates using OLS are the best linear unbiased estimators (BLUE). The use of portfolio returns to measure market risk substantially lowers the probability of measurement error but other violations of the CLR assumptions have varied and possibly substantial impacts on the validity of a given model. Thus, tests for violations of the OLS assumptions are conducted on the constant risk and the varying risk models.

MULTICOLLINEARITY

One of the principle assumptions of the classical linear regression model is that there is no perfectly linear relationship between any set of the independent variables in a regression model; a linear relationship implies multicollinearity. OLS estimation in the presence of multicollinearity is unbiased but interpretation of the parameter estimates is difficult and large parameter estimate variances occur. Two tests for multicollinearity are conducted: the calculation of variance inflation factors and condition number analysis.

The diagonal elements of the variance-covariance matrix are the variance inflation factors (VIFs). A VIF less than 10 is indicated by Belsley, Kuh, and Welsch (1980) to be a sound indication that no significant multicollinearity exists. Since multicollinearity measures collinear relationships between independent variables, the multicollinearity diagnostic statistics will be the same for all the portfolios (all the time–series regressions with the same number of observations) and the VIF is not applicable for the constant risk model with one independent variable other than the intercept. For the full period sample varying risk model examined, the VIFs are 1.719, 2.647, and 2.054 for the D_{1t}, R_{mt}, and $R_{mt}D_{1t}$ variables, respectively. Thus, the VIF statistics indicate a low level of multicollinearity.

Condition number analysis allows for identification of multicollinearity between two or more variables. Tests of multicollinearity based on simple correlation analysis cannot indicate significant multicollinearity between groups of three or more independent variables. Belsley, Kuh, and Welsch (1980) again indicate a value of 10 or less for the condition number index as a sound indication of the lack of serious multicollinear relationships. The results of the condition number analysis also indicate a lack of any substantial collinear relationships between the independent variables based, in part, on the condition numbers which are 1.413, 2.774, and 4.2884 for the D_{1t}, R_{mt}, and $R_{mt}D_{1t}$ variables, respectively, for the full period sample; and further examinations of the condition number results.

AUTOCORRELATION

Correlation between error terms is precluded in the CLR framework. Statistical inferences based on autocorrelated data are undependable due to biased standard errors. The Durbin-Watson test (DW) is used to test for first-order serial correlation of the error terms. For the constant risk model, based on one independent variable other than the intercept, a DW statistic of approximately 2.316 or greater indicates the possibility (at a .01-level) of negative serial (auto) correlation while a DW statistic of approximately 1.684 or less indicates the possibility (at a .01-level) of positive serial (auto) correlation. The relevant DW statistics for the varying risk model are 2.296 for negative serial correlation and 1.715 for positive serial correlation. The DW statistics in Table 3 provide no evidence of significant autocorrelation in either the constant or varying risk model.

SUBPERIODS OF THE MODEL

The time series of returns for each portfolio are separated into early period regressions (January, 1926 - June, 1957; 390 observations) and late period regressions (July, 1957 - December, 1990; 390 observations) to further test the robustness of the varying risk model. The median market return for the early period was 1.5365% while the median market return for the late period was 1.2085%. Condition number analysis indicates again no substantial multicollinearity for the two sub-periods. Condition numbers for the early period are 1.393, 2.678, and 4.055 for the D_{1t}, R_{mt}, and $R_{mt}D_{1t}$ variables, respectively; and for the late period, they are 1.452, 3.009, and 5.650 for the D_{1t}, R_{mt}, and $R_{mt}D_{1t}$ variables, respectively.

The largest VIF for either period model is 4.452. The DW statistics in Tables 4 and 5 also provide no evidence of significant autocorrelation.

The pattern of declining beta size as the size of the portfolio increases for the constant risk market model is still evident for both the early and late period regressions. The pattern of significantly larger bull market betas for small market value portfolios and significantly smaller bull market betas for large market value portfolios is maintained for both the early and late period samples. The pattern of larger bull market betas switches to that of smaller bull market betas at the MV6 portfolio for the early and full data period samples but at the MV8 portfolio for the late period sample. There is a distinct pattern of fewer statistically significant differences between the constant risk and varying risk models for the late period as compared to the full period and early period regressions.

{Insert Tables 4 and 5}

TIME-VARYING AND CROSS-SECTIONAL HETEROSKEDASTICITY

The third assumption of the CLR is that the error terms in a regression model have a constant variance, the error terms are normally distributed, and that the error terms are uncorrelated. If the error variance is not constant, the disturbances are said to be heteroskedastic. The parameter estimates are unbiased but the standard errors are biased (as well as inefficient) and valid statistical inferences such as the difference between the constant and varying risk model are invalid.

White's test is a general test for heteroskedasticity, based on the relationship of a regression of the squared residuals of the original regression on a constant and all the unique variables of the cross-product of the independent variables. The p-values for White's test shown in Tables 3, 4, and 5 are based on corollary 1 of White (1980). Another similar heteroskedasticity diagnostic is the modified Breusch-Pagan (BP) test.[3] The BP test statistics in the tables are based on all the independent variables in the specific regression including the intercept. Results of both tests in Tables 3, 4, and 5 indicate a significant level of heteroskedasticity in all the regressions.

A more specific diagnostic for cross-sectional heteroskedasticity is also conducted. The differences in standard deviations between bull and bear markets, especially for small market value portfolios, indicate the possibility that the error variances may differ between bull and bear markets. T-tests of the means of the squared residuals of the full sample regressions are conducted where the means of the bull market squared residuals are tested against the means of the bear market squared residuals. The results of the t-tests are indicated in Table 6. For all portfolios, except MV5, the t-tests indicate highly significant differences in squared residuals between bull and bear markets indicating substantial cross-sectional heteroskedasticity, based on bull vs. bear markets. The same t-tests for the early and late periods (not shown in a table) are also highly significant except for MV5 and MV6 (early period) and MV13 (late period) portfolios.

{Insert Table 6}

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[3]The modified Breusch-Pagan test (Greene, 1993) is less sensitive to an assumption of normality than the original test.

The substantially larger bull market squared residuals indicate a higher degree of uncertainty in forecasting during bull markets, which, in turn indicates a higher degree of informational asymmetry in bull markets. The residuals in both bull and bear markets for the four smallest portfolios (MV1-MV4) are, for the most part, larger than the residuals of the other portfolios. Thus, evidence is provided for higher informational asymmetries as a contributor to higher market betas for small market value portfolios. The fact that larger market value portfolios have higher betas in bear markets, however, is contra-indicated, following the same logic, by their larger squared residuals in bull markets.

A further set of regressions was estimated for the full period data sample with the error variance modeled following equation (3):

(3) $h_t = \omega_1 + \kappa_1 D_{1t}$

where,

$\varepsilon_t = (h_t)^{1/2} e_t$, and

h_t = the conditional variance of the error term of the return-generating process,

ω_1 = an intercept term,

D_{1t} = a dummy variable equal to 1 for bear months and 0 for bull months,

ε_t = the unconditional variance of the error term of the return-generating process[4], and

e_t = a residual sequence, which is normally and independently distributed with zero mean and unit variance.

[4] ε_t is assumed to follow a white noise process with zero mean, a constant variance, and zero autocorrelation of then first and second moments.

Since the parameter estimate for D_{1t} was always highly significant and negative, substantial cross-sectional heteroskedasticity based on bull vs. bear markets is again indicated.

A regression model may also have a conditional error variance that is based on past residuals. Engle Lagrange multiplier tests (Engle, 1982) are based on regressing the squared residual of the original regression on lagged squared residuals of the original regression. The test statistic is the sample size T times the uncentered R^2 from the residual squared regression. The test statistic is distributed as a χ^2 variable with m-degrees of freedom, where m is the largest lag length of the lagged squared residuals. For the full period model, the Engle Lagrange multiplier tests, for all portfolios and for both the constant and varying risk models, indicate significant long-memory (long-lagged) time varying heteroskedasticity. Thus, the equation (4) GARCH(1,1) model with an AR(1) component is estimated, based on the principle of parsimony, to account for the time-varying heteroskedasticity, with the results presented in Table 7:[5]

(4) $h_t = \omega_2 + \alpha_2\varepsilon^2_{t-1} + \gamma_2 h_{t-1}$, and

$\nu_t = \varepsilon_t - \varphi_1\nu_{t-1}$

where the definitions of the variables follow equation (3) except, ν_{t-1} is the AR1 term, ε^2_{t-1} is the ARCH1 term, and h_{t-1} is the GARCH1 term. The inclusion of the AR(1) term allows for a further check of first-order autocorrelation.

{Insert Table 7}

[5]The MV1, MV5, MV10, MV15, and MV20 AR(1)-GARCH(1,1) regressions are estimated with a number of starting

Time-varying heteroskedasticity does not appear to compromise the validity of the statistical tests of significance for the varying risk model. The b_{bull} and b_{bear}-b_{bull} parameter estimates are all still highly significant. The ARCH 1 and GARCH1 components are also always highly significant in the full sample regressions. The AR(1) terms are not significant for either model except for the MV19 and MV20 portfolios of the constant risk model. The GARCH parameter estimate is always large indicating a long-memory process with all past residuals influential in determining the current error variance. This behavior can be due to the clustering of bull market returns around business cycle troughs and bear market returns around business-cycle peaks as indicate in Table 1 (p. 273) of Bhardwaj and Brooks.

For the early subperiod sample, the Lagrange multiplier tests indicate time-varying heteroskedasticity for both the constant and varying risk models except for the MV1 portfolio (both models) and the MV17 portfolio (constant risk model). The Lagrange multiplier tests are less conclusive for the late subperiod. Six of the portfolios exhibit few significant parameters for both models for the late period while several more of the portfolios do not exhibit many significant parameters for the varying risk model.

A relevant question is if the cross-sectional bull/bear market heteroskedasticity will still be significant when time-varying heteroskedasticity is explicitly modeled. Equation (5) is estimated to test this question:

(5) $$h_t = \omega_3 + \alpha_3\varepsilon^2_{t-1} + \gamma_3 h_{t-1} + \kappa_3 D_{1t}$$

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values to test for global maximization. In all cases, the estimates were stable to different starting values.

where definitions of the variables follow from equations (3) and (4). When time-varying heteroskedasticity is modeled, the D_{1t} parameter (η_1) loses all significance in all regressions and the parameter estimate is negligible. This result is not surprising since bull and bear market returns are clustered, respectively, together.

Examination of equation (5) estimates for the early and late periods indicates similar patterns of significant large parameter estimates for the ARCH1 and GARCH1 parameters except for the larger portfolios in the late period; and insignificant and negligible estimates for κ_3.

SUMMARY AND CONCLUSIONS

The ability of the varying risk market model to capture differences in abnormal returns and systematic risk between bull and bear markets has been considered in this study. The model's statistical robustness has been examined. Examinations of its robustness to multicollinearity, and autocorrelation produce robust results. Examination of the varying risk model is also conducted over two equal subperiods. F-tests indicate significant difference between the constant risk and varying risk models in twenty out of twenty cases for the OLS full period regressions; nineteen out of twenty cases for the early period OLS regressions; but only fifteen out of twenty cases for the late period OLS regressions; and nineteen cases for the full period AR(1)-GARCH(1,1) regressions.

Of concern, is the varying risk model's robustness to heteroskedasticity. Several tests indicate the presence of heteroskedasticity. Modeling the heteroskedastic error terms using GARCH methodology indicates that the varying risk model is robust to the heteroskedastic conditions identified.

A truncated sample using recent data, where financially distressed firms are identified through balance sheet data, should be used to test if the financially distressed firms affect the empirical results of the varying risk model. More recent data would have to be used due to the difficulty in obtaining complete balance sheet data for the earlier subperiod of this study. Various balance sheet measures such as sales expenses divided by total assets and research and development expenses divided by total assets as well as other measures of informational asymmetry may also be used in cross-sectional regressions using the market value portfolio regressions results to see if the small market value firms exhibit greater informational asymmetries which affect their market beta estimates. However, these tests may be of limited use as one measure of informational asymmetries, which would confound tests using other measures of informational asymmetries used, is firm size.

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Table 1 – Summary of Securities Retained Each Year of the Study

Year	Sample Size	Year	Sample Size	Year	Sample Size
1926	405	1948	850	1970	2203
1927	382	1949	874	1971	2309
1928	413	1950	811	1972	2417
1929	518	1951	865	1973	2561
1930	555	1952	922	1974	2540
1931	558	1953	974	1975	2508
1932	505	1954	978	1976	2110
1933	387	1955	989	1977	2363
1934	450	1956	972	1978	2293
1935	537	1957	993	1979	2258
1936	638	1958	1009	1980	2219
1937	348	1959	1018	1981	2167
1938	609	1960	1032	1982	2152
1939	590	1961	1058	1983	2113
1940	633	1962	1093	1984	2113
1941	684	1963[a]	1825	1985	2069
1942	739	1964	1878	1986	2053
1943	687	1965	1921	1987	2126
1944	682	1966	1961	1988	2211
1945	653	1967	1999	1989	2277
1946	729	1968	1991	1990	2307
1947	824	1969	2059		

[a]Starting in 1963, AMEX stocks are included in the portfolios.

Table 2 - Average Percentage Monthly Returns and Standard Deviations of Returns in All, Bull, and Bear Months; and Summary of Portfolio Market Values (in $ millions) from 1926-1990

	All Months		Bear Months (390)		Bull Months (390)			
Portfolio (by Mkt. Value)	Average Return	Standard Deviation	Average Return	Standard Deviation	Average Return	Standard Deviation	Range of Market Values	Avg. Mkt. Value
MV1	2.65%	14.25%	-4.71%	6.89%	10.00%	15.83%	.174-4.56	2.51
MV2	1.72%	11.73%	-4.76%	6.61%	8.20%	12.15%	.371-11.22	4.81
MV3	1.47%	10.59%	-4.80%	6.40%	7.74%	10.25%	.704-20.00	7.07
MV4	1.38%	10.18%	-4.71%	6.49%	7.47%	9.53%	1.18-32.37	9.72
MV5	1.18%	9.32%	-4.56%	5.93%	6.91%	8.54%	1.58-44.47	12.75
MV6	1.22%	8.80%	-4.41%	6.09%	6.85%	7.38%	2.08-58.46	16.34
MV7	1.24%	8.46%	-4.23%	5.79%	6.70%	7.08%	2.81-77.99	20.57
MV8	1.18%	8.29%	-4.22%	5.70%	6.57%	6.85%	3.63-98.56	25.59
MV9	1.19%	8.33%	-4.16%	5.72%	6.53%	7.00%	4.46-127.90	31.71
MV10	1.17%	8.46%	-4.14%	5.72%	6.47%	7.36%	5.89-165.13	39.74
MV11	1.17%	8.30%	-4.09%	5.67%	6.43%	7.10%	8.19-220.06	50.1
MV12	1.21%	7.72%	-3.83%	5.38%	6.25%	6.28%	11.16-295.05	63.53
MV13	1.15%	7.52%	-3.71%	5.38%	6.01%	6.08%	13.54-390.70	81.86
MV14	1.01%	7.14%	-3.65%	5.29%	5.68%	5.52%	17.89-550.73	107.01
MV15	0.94%	7.01%	-3.56%	5.20%	5.44%	5.55%	23.52-780.52	142.5
MV16	1.06%	6.94%	-3.40%	5.07%	5.52%	5.55%	32.61-1,109.76	196.03
MV17	1.01%	6.86%	-3.35%	5.04%	5.38%	5.52%	50.30-1,694.09	279.14
MV18	1.05%	6.52%	-3.14%	4.77%	5.24%	5.23%	78.71-2,670.64	421.71
MV19	0.88%	5.79%	-2.78%	4.41%	4.54%	4.58%	144.98-4,839.56	679.42
MV20	0.87%	5.58%	-2.57%	4.37%	4.33%	4.41%	425.49-35,486.9	2,601.33
Market[a]	1.28%	7.88%	-3.79%	5.10%	6.35%	6.84%		

[a]CRSP equally-weighted return index with dividends included (EWI)

Table 3 - Full Data Sample (OLS estimation; 780 observations)

Constant Risk Market Model

	a_1	b_1	Adj. R^2	White's[a]	Br-P[a]	D-W[b]	F-stat[c]
MV1	0.007908 *	1.590590 ***	.7744	.0001	.0001	2.033	65.6660 ***
MV2	0.000525	1.392440 ***	.8759	.0001	.0001	2.046	64.3756 ***
MV3	-0.000872	1.282850 ***	.9108	.0001	.0001	1.990	21.5115 ***
MV4	-0.001517 *	1.255760 ***	.9458	.0001	.0001	1.890	18.5671 ***
MV5	-0.002552 ***	1.153340 ***	.9499	.0001	.0001	2.057	10.2161 ***
MV6	-0.001539 **	1.092100 ***	.9558	.0001	.0001	1.961	12.8767 ***
MV7	-0.000909	1.047160 ***	.9506	.0001	.0001	1.905	16.8083 ***
MV8	-0.001332 **	1.028330 ***	.9541	.0001	.0001	2.011	25.5575 ***
MV9	-0.001255 *	1.031890 ***	.9524	.0001	.0001	2.308	17.0896 ***
MV10	-0.001589 **	1.044560 ***	.9464	.0001	.0001	2.135	4.3225 **
MV11	-0.001333 *	1.022080 ***	.9411	.0001	.0001	2.092	7.4342 ***
MV12	-0.000140	0.945210 ***	.9315	.0001	.0001	2.001	36.6536 ***
MV13	-0.000561	0.921880 ***	.9334	.0001	.0001	2.035	37.9891 ***
MV14	-0.001346 *	0.865770 ***	.9128	.0001	.0001	2.043	77.6867 ***
MV15	-0.001874 **	0.841730 ***	.8952	.0001	.0001	2.050	52.9928 ***
MV16	-0.000586	0.833530 ***	.8947	.0001	.0001	2.057	42.1231 ***
MV17	-0.000847	0.814100 ***	.8756	.0001	.0001	1.970	39.4700 ***
MV18	-0.000109	0.774420 ***	.8744	.0001	.0001	2.024	33.6579 ***
MV19	-0.000756	0.669680 ***	.8308	.0001	.0001	1.940	45.6186 ***
MV20	-0.000312	0.621260 ***	.7682	.0001	.0001	2.007	42.8428 ***

Table 3 - Full Data Sample (continued)

Varying Risk Market Model

	a_{bull}	a_{bear}[d]	a_{bear}-a_{bull}	b_{bull}	b_{bear}[d]	b_{bear}-b_{bull}	Adj. R^2	White's[a]	Br-P[a]	D-W[b]
MV1	-0.023167 ***	-0.002914 *	0.020253 ***	1.984850 ***	1.152910 **	-0.831940 ***	.8065	.0001	.0001	2.085
MV2	-0.020396 ***	-0.002757 *	0.017639 ***	1.640770 ***	1.170000 **	-0.470770 ***	.8933	.0001	.0001	2.118
MV3	-0.010203 ***	-0.003073	0.007130 **	1.396760 ***	1.170350 **	-0.226410 ***	.9152	.0001	.0001	2.008
MV4	-0.009266 ***	-0.000421	0.008845 ***	1.337870 ***	1.215090 **	-0.122780 ***	.9482	.0001	.0001	1.961
MV5	-0.006895 ***	-0.003498	0.003397 *	1.206020 ***	1.102400 **	-0.103620 ***	.9511	.0001	.0001	2.070
MV6	0.002457 **	-0.000133	-0.002590	1.041340 ***	1.148640 **	0.107300 ***	.9571	.0001	.0001	1.973
MV7	0.004349 ***	-0.000298	-0.004647 ***	0.985640 ***	1.099220 **	0.113580 ***	.9525	.0001	.0001	1.924
MV8	0.004797 ***	-0.000752	-0.005549 ***	0.957210 ***	1.086610 **	0.129400 ***	.9568	.0001	.0001	2.008
MV9	0.003447 ***	0.000034	-0.003413 **	0.973720 ***	1.091840 **	0.118120 ***	.9543	.0001	.0001	2.304
MV10	0.000048	0.000215	0.000167	1.018540 ***	1.089860 **	0.071320 ***	.9468	.0001	.0001	2.122
MV11	0.002005	-0.000175	-0.002180	0.979750 ***	1.069010 **	0.089260 ***	.9421	.0001	.0001	2.133
MV12	0.008142 ***	0.000533 *	-0.007609 ***	0.849570 ***	1.021950 **	0.172380 ***	.9373	.0001	.0001	2.051
MV13	0.006799 ***	0.001352 *	-0.005447 ***	0.831280 ***	1.013810 **	0.182530 ***	.9391	.0001	.0001	2.034
MV14	0.009627 ***	0.001434 *	-0.008193 ***	0.731000 ***	1.001530 **	0.270530 ***	.9272	.0001	.0001	2.195
MV15	0.007858 ***	0.001103 *	-0.006755 ***	0.720020 ***	0.971360 **	0.251340 ***	.9076	.0001	.0001	2.128
MV16	0.008570 ***	0.001424 *	-0.007146 ***	0.722400 ***	0.941220 **	0.218820 ***	.9048	.0001	.0001	2.112
MV17	0.008559 ***	0.001456 *	-0.007103 ***	0.698920 ***	0.929030 **	0.230110 ***	.8868	.0001	.0001	2.042
MV18	0.008944 ***	0.000763	-0.008181 ***	0.669310 ***	0.860780 **	0.191470 ***	.8842	.0001	.0001	2.055
MV19	0.009327 ***	0.001360 *	-0.007967 ***	0.547720 ***	0.786500 **	0.238780 ***	.8482	.0001	.0001	2.045
MV20	0.010900 ***	0.001776 *	-0.009124 ***	0.486770 ***	0.746380 **	0.259610 ***	.7907	.0001	.0001	2.123

[a]White's test for heteroskedasticity. Br-P is the modified Breusch-Pagan test for heteroskedasticity (Greene, 1993).
[b]The Durbin-Watson test for first-order autocorrelation of the error terms. A value near 2 indicates no serial first-order autocorrelation.
[c]The F-statistic is a test of the joint hypothesis that $a_{bear}=a_{bull}$, and $b_{bear}=b_{bull}$. The test statistic is defined as F={(SSE1 - SSE2)/k}/{SSE2/(n-2k)}, distributed with F(k,n-2k) and where SSE1 and SSE2 are, respectively, the sum of squared errors from the constant risk and varying risk market models; n is the total number of return observations in both the bull and bear markets; and k is the number of parameters estimated in the constant risk market model.
[d]The significance of the a_{bear} and b_{bear} parameter estimates is tested using an F-statistic based on unrestricted and restricted regressions.

*** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

Table 4 - Early Period Sample (monthly observations from January, 1926 to June, 1957; OLS estimation; 390 observations)

Constant Risk Market Model

	a_1	b_1	Adj. R^2	White's[a]	Br-P[a]	D-W[b]	F-stat[c]
MV1	0.009364**	1.67235***	.7850	.0001	.0001	2.064	32.2170***
MV2	-0.000399	1.44873***	.8866	.0001	.0001	2.140	32.9229***
MV3	-0.001040	1.31357***	.9156	.0001	.0001	2.098	9.0266***
MV4	-0.002589*	1.28115***	.9549	.0001	.0001	1.913	5.3423***
MV5	-0.003499***	1.15155***	.9529	.0001	.0003	2.082	4.8023***
MV6	-0.001969*	1.08979***	.9584	.0001	.0001	1.963	13.6637***
MV7	-0.001945*	1.03038***	.9477	.0001	.0001	1.856	12.1846***
MV8	-0.001482	1.01701***	.9499	.0001	.0001	2.052	19.0677***
MV9	-0.000884	1.01482***	.9450	.0001	.0001	2.279	11.0229***
MV10	-0.001717	1.04002***	.9390	.0001	.0001	2.192	2.1018
MV11	-0.001232	1.02905***	.9340	.0001	.0001	2.163	3.7189**
MV12	0.000271	0.92864***	.9233	.0001	.0001	2.030	23.4420***
MV13	-0.000972	0.91495***	.9303	.0001	.0001	2.171	18.3654***
MV14	-0.002094	0.84946***	.9035	.0001	.0001	2.051	44.1860***
MV15	-0.003467**	0.82919***	.8874	.0001	.0001	2.116	31.2920***
MV16	-0.000807	0.82238***	.8997	.0001	.0001	2.142	25.1311***
MV17	-0.001150	0.81282***	.8763	.0001	.0001	1.999	27.7203***
MV18	-0.000076	0.78143***	.8937	.0001	.0001	2.102	25.9198***
MV19	-0.001210	0.65827***	.8585	.0001	.0001	1.958	31.7071***
MV20	-0.000090	0.62406***	.8173	.0001	.0001	1.984	37.3459***

Table 4 - Early Period Sample (continued)

Varying Risk Market Model

	a_{bull}	a_{bear}[d]	a_{bear}-a_{bull}	b_{bull}	b_{bear}[d]	b_{bear}-b_{bull}	Adj. R^2	White's[a]	Br-P[a]	D-W[b]
MV1	-0.021197 ***	-0.009433	0.011764	2.01278 ***	1.13320 *	-0.87958 ***	.8148	.0001	.0001	2.066
MV2	-0.021441 ***	-0.008167	0.013274 **	1.66490 ***	1.16135 **	-0.50355 ***	.9026	.0001	.0001	2.177
MV3	-0.007293 **	-0.007425	-0.000132	1.39216 ***	1.16213 **	-0.23003 ***	.9190	.0001	.0001	2.109
MV4	-0.008683 ***	-0.00222	0.006463 *	1.33453 ***	1.24035 **	-0.09418 **	.9559	.0001	.0001	1.950
MV5	-0.007748 ***	-0.005058	0.002690	1.19517 ***	1.09368 **	-0.10149 ***	.9538	.0001	.0001	2.104
MV6	0.004857 **	-0.000581	-0.005438 **	1.02364 ***	1.16469 **	0.14105 ***	.9609	.0001	.0001	1.991
MV7	0.005408 ***	-0.001342	-0.006750 **	0.96225 ***	1.09661 **	0.13436 ***	.9506	.0001	.0001	1.876
MV8	0.007297 ***	-0.000858	-0.008155 ***	0.93601 ***	1.09453 **	0.15852 ***	.9542	.0001	.0001	2.016
MV9	0.004978 **	0.001603	-0.003375	0.95346 ***	1.10012 **	0.14666 ***	.9477	.0001	.0001	2.282
MV10	-0.000012	0.000665	0.000677	1.01632 ***	1.09173 **	0.07541 **	.9394	.0001	.0001	2.178
MV11	0.002104	0.001061	-0.001043	0.99104 ***	1.09181 **	0.10077 ***	.9349	.0001	.0001	2.204
MV12	0.011364 ***	0.000971	-0.010393 ***	0.82663 ***	1.02512 **	0.19849 ***	.9312	.0001	.0001	2.065
MV13	0.006305 ***	0.002672	-0.003633	0.83681 ***	1.02987 **	0.19306 ***	.9360	.0001	.0001	2.128
MV14	0.011851 ***	0.000849	-0.011002 ***	0.71395 ***	1.00418 **	0.29023 ***	.9211	.0001	.0001	2.213
MV15	0.008051 ***	0.001161	-0.006890 **	0.70953 ***	0.99261 **	0.28308 ***	.9026	.0001	.0001	2.211
MV16	0.009130 ***	0.002824	-0.006306 **	0.72042 ***	0.95749 **	0.23707 ***	.9108	.0001	.0001	2.204
MV17	0.010092 ***	0.003459	-0.006633 *	0.69569 ***	0.97382 **	0.27813 ***	.8913	.0001	.0001	2.067
MV18	0.010945 ***	0.001826	-0.009119 ***	0.67581 ***	0.89686 **	0.22105 ***	.9058	.0001	.0001	2.166
MV19	0.009607 ***	0.002539	-0.007068 **	0.54800 ***	0.80204 **	0.25404 ***	.8779	.0001	.0001	2.075
MV20	0.013534 ***	0.003146	-0.010388 ***	0.49040 ***	0.78106 **	0.29066 ***	.8461	.0001	.0001	2.187

[a]White's test for heteroskedasticity. Br-P is the modified Breusch-Pagan test for heteroskedasticity (Greene, 1993).
[b]The Durbin-Watson test for first-order autocorrelation of the error terms. A value near 2 indicates no serial first-order autocorrelation.
[c]The F-statistic is a test of the joint hypothesis that $a_{bear}=a_{bull}$, and $b_{bear}=b_{bull}$. The test statistic is defined as F={(SSE1 - SSE2)/k}/{SSE2/(n-2k)}, distributed with F(k,n-2k) and where SSE1 and SSE2 are, respectively, the sum of squared errors from the constant risk and varying risk market models; n is the total number of return observations in both the bull and bear markets; and k is the number of parameters estimated in the constant risk market model.
[d]The significance of the a_{bear} and b_{bear} parameter estimates is tested using an F-statistic based on unrestricted and restricted regressions.

*** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

Table 5 - Late Period Sample (from July, 1957 to December, 1990; OLS estimation; 390 observations)

Constant Risk Market Model

	a_1	b_1	Adj. R^2	White's[a]	Br-P[a]	D-W[b]	F-stat[c]
MV1	0.006882***	1.365200***	.7578	.0001	.0001	1.977	18.5139***
MV2	0.001722	1.240890***	.8542	.0001	.0001	1.956	15.3305***
MV3	-0.000550	1.199520***	.9002	.0001	.0001	1.955	9.4257***
MV4	-0.000331	1.188650***	.9222	.0001	.0001	1.833	13.3045***
MV5	-0.001626*	1.160000***	.9422	.0001	.0001	1.962	11.7553***
MV6	-0.001125	1.099220***	.9489	.0023	.0142	2.040	1.8409
MV7	0.000030	1.094810***	.9604	.0047	.0016	2.059	1.0870
MV8	-0.001240**	1.059450***	.9654	.0019	.0004	1.833	0.1148
MV9	-0.001708***	1.077690***	.9725	.0104	.0029	2.159	0.0206
MV10	-0.001484**	1.057170***	.9658	.0001	.0001	1.868	1.8783
MV11	-0.001398**	1.002900***	.9618	.0001	.0001	1.989	12.3352***
MV12	-0.000629	0.989550***	.9538	.0008	.0004	1.993	2.8853*
MV13	-0.000189	0.941500***	.9414	.0001	.0001	1.779	20.4478***
MV14	-0.000689	0.911610***	.9387	.0001	.0001	1.931	16.4307***
MV15	-0.000364	0.878880***	.9185	.0001	.0001	1.939	10.3206***
MV16	-0.000424	0.864330***	.8835	.0001	.0001	1.947	11.8562***
MV17	-0.000554	0.818160***	.8728	.0001	.0001	1.896	8.2797***
MV18	-0.000105	0.755280***	.8226	.0001	.0001	1.962	12.5556***
MV19	-0.000365	0.701620***	.7729	.0001	.0001	1.960	11.8945***
MV20	-0.000516	0.613210***	.6560	.0001	.0001	2.062	9.8255***

Table 5 - Late Period Sample (continued)

Varying Risk Market Model

	a_{bull}	a_{bear} [d]	a_{bear}-a_{bull}	b_{bull}	b_{bear} [d]	b_{bear}-b_{bull}	Adj. R^2	White's[a]	Br-P[a]	D-W[b]
MV1	-0.017039 ***	0.003295	0.020334 ***	1.779570 ***	1.165170 **	-0.614400 ***	.7779	.0001	.0001	2.047
MV2	-0.013712 ***	0.001200	0.014912 ***	1.498040 ***	1.147880 **	-0.350160 ***	.8643	.0001	.0001	2.108
MV3	-0.009996 ***	-0.001134	0.008862 ***	1.358410 ***	1.137260 **	-0.221150 ***	.9044	.0001	.0001	1.960
MV4	-0.010334 ***	0.000033	0.010367 ***	1.351320 ***	1.142490 **	-0.208830 ***	.9268	.0001	.0001	1.859
MV5	-0.009327 ***	-0.001748	0.007579 ***	1.287530 ***	1.116350 **	-0.171180 ***	.9452	.0001	.0001	1.957
MV6	-0.003539 **	0.000711	0.004250 *	1.128530 ***	1.123230 **	-0.005300	.9491	.0001	.0008	2.031
MV7	-0.001823	0.000979	0.002802	1.119920 ***	1.104000 **	-0.015920	.9604	.0008	.0015	2.062
MV8	-0.000995	-0.000907	0.000088	1.053520 ***	1.067460 **	0.013940	.9652	.0001	.0002	1.833
MV9	-0.001616	-0.001580	0.000036	1.075440 ***	1.080760 **	0.005320	.9723	.0058	.0021	2.158
MV10	0.000520	-0.001107	-0.001627	1.022030 ***	1.075460 **	0.053430 *	.9660	.0001	.0001	1.878
MV11	0.004250 ***	-0.001800	-0.006050 ***	0.912190 ***	1.025010 **	0.112820 ***	.9640	.0006	.0056	2.101
MV12	0.002030	-0.000041	-0.002071	0.942410 ***	1.015580 **	0.073170 **	.9543	.0025	.0058	2.107
MV13	0.008003 ***	-0.000200	-0.008203 ***	0.806650 ***	0.985090 **	0.178440 ***	.9467	.0001	.0001	1.865
MV14	0.005409 ***	0.001374	-0.004035 **	0.799440 ***	0.985700 **	0.186260 ***	.9433	.0012	.0003	2.014
MV15	0.006484 ***	-0.001010	-0.007494 ***	0.769780 ***	0.902510 **	0.132730 ***	.9222	.0001	.0001	1.969
MV16	0.008126 ***	-0.000552	-0.008678 ***	0.724270 ***	0.907450 **	0.183180 ***	.8897	.0001	.0001	1.974
MV17	0.006878 ***	-0.001633	-0.008511 ***	0.701900 ***	0.836210 **	0.134310 ***	.8774	.0001	.0001	1.937
MV18	0.010105 ***	-0.001854	-0.011959 ***	0.597080 ***	0.774710 **	0.177630 ***	.8326	.0001	.0001	1.996
MV19	0.010226 ***	-0.001199	-0.011425 ***	0.531930 ***	0.741490 **	0.209560 ***	.7849	.0001	.0001	1.998
MV20	0.010386 ***	-0.000286	-0.010672 ***	0.432340 ***	0.676140 *	0.243800 ***	.6710	.0003	.0001	2.095

[a]White's test for heteroskedasticity. Br-P is the modified Breusch-Pagan test for heteroskedasticity (Greene, 1993).
[b]The Durbin-Watson test for first-order autocorrelation of the error terms. A value near 2 indicates no serial first-order autocorrelation.
[c]The F-statistic is a test of the joint hypothesis that $a_{bear}=a_{bull}$, and $b_{bear}=b_{bull}$. The test statistic is defined as F={(SSE1 - SSE2)/k}/{SSE2/(n-2k)}, distributed with F(k,n-2k) and where SSE1 and SSE2 are, respectively, the sum of squared errors from the constant risk and varying risk market models; n is the total number of return observations in both the bull and bear markets; and k is the number of parameters estimated in the constant risk market model.
[d]The significance of the a_{bear} and b_{bear} parameter estimates is tested using an F-statistic based on unrestricted and restricted regressions.

*** Significant at the 1 percent level
** Significant at the 5 percent level
* Significant at the 10 percent level

Table 6 - A Comparison of Bull and Bear Market Squared Residuals from the Full Sample (1926-1990) Varying Risk Market Model Regressions, Based on T-tests of the Mean of Bull and Bear Market Squared Residuals

		Mean[a]	Std. Dev.[b]	N	Prob > |T|[c]
MV1	Bull	0.6574	2.6013	390	0.0001
	Bear	0.1265	0.2560	390	
MV2	Bull	0.2143	0.8401	390	0.0025
	Bear	0.0787	0.2694	390	
MV3	Bull	0.1382	0.5034	390	0.0010
	Bear	0.0518	0.1072	390	
MV4	Bull	0.0708	0.1752	390	0.0004
	Bear	0.0364	0.0771	390	
MV5	Bull	0.0497	0.1124	390	0.1276
	Bear	0.0353	0.1493	390	
MV6	Bull	0.0391	0.1013	390	0.0435
	Bear	0.0273	0.0557	390	
MV7	Bull	0.0475	0.1326	390	0.0001
	Bear	0.0204	0.0380	390	
MV8	Bull	0.0414	0.1661	390	0.0077
	Bear	0.0180	0.0467	390	
MV9	Bull	0.0454	0.1335	390	0.0001
	Bear	0.0180	0.0444	390	
MV10	Bull	0.0570	0.3009	390	0.0160
	Bear	0.0191	0.0722	390	
MV11	Bull	0.0559	0.3306	390	0.0641
	Bear	0.0239	0.0832	390	
MV12	Bull	0.0569	0.2114	390	0.0004
	Bear	0.0177	0.0446	390	
MV13	Bull	0.0467	0.1464	390	0.0017
	Bear	0.0221	0.0476	390	
MV14	Bull	0.0559	0.2265	390	0.0013
	Bear	0.0183	0.0365	390	
MV15	Bull	0.0660	0.2926	390	0.0069
	Bear	0.0248	0.0661	390	
MV16	Bull	0.0647	0.2219	390	0.0011
	Bear	0.0270	0.0473	390	
MV17	Bull	0.0769	0.3697	390	0.0129
	Bear	0.0294	0.0679	390	
MV18	Bull	0.0647	0.1688	390	0.0010
	Bear	0.0339	0.0721	390	
MV19	Bull	0.0687	0.2087	390	0.0015
	Bear	0.0329	0.0716	390	
MV20	Bull	0.0846	0.2293	390	0.0020
	Bear	0.0458	0.0906	390	

[a]Parameter values are all times 10^{-2}.
[b]Parameter values are all times 10^{-2}.
[c]p-values are based on the assumption of unequal variances of the means. For all twenty portfolios, the variances of the means for bull and bear markets were significantly different at a .0001-level.

Table 7 - GARCH Regressions (maximum likelihood estimation of an AR(1)-GARCH(1,1) model; 780 observations)

Constant Risk Model

	a1	b1	AR1	ARCH0	ARCH1	GARCH1	Total R^{2a}	LIKLHD[b]	F-stat[c]
MV1	0.005154 ***	1.344900 ***	-0.040750	0.000010	0.098030 ***	0.911240 ***	0.7546	1218.93	102.878 ***
MV2	0.001513	1.255550 ***	0.016170	0.000016 **	0.114930 ***	0.882430 ***	0.8676	1548.12	88.950 ***
MV3	-0.001135	1.205570 ***	-0.044680	0.000004 **	0.078080 ***	0.921530 ***	0.9074	1736.54	35.802 ***
MV4	-0.000356	1.210840 ***	-0.065960	0.000010 **	0.108030 ***	0.877510 ***	0.9449	1911.28	23.553 ***
MV5	-0.001513 **	1.147870 ***	0.011340	0.000006 ***	0.084860 ***	0.903900 ***	0.9499	2003.08	9.045 ***
MV6	-0.000961 *	1.120450 ***	-0.044820	0.000010 ***	0.106810 ***	0.866640 ***	0.9552	2084.22	5.554 ***
MV7	-0.000477	1.082350 ***	-0.004150	0.000003 *	0.079280 ***	0.914410 ***	0.9495	2139.23	5.371 ***
MV8	-0.001586 ***	1.052190 ***	-0.036140	0.000004 **	0.130710 ***	0.865490 ***	0.9536	2194.49	27.455 ***
MV9	-0.001301 ***	1.067360 ***	0.048540	0.000002 **	0.091560 ***	0.901970 ***	0.9520	2239.88	13.275 ***
MV10	-0.001040 **	1.060610 ***	-0.039550	0.000003 **	0.103530 ***	0.894110 ***	0.9457	2181.66	3.761 ***
MV11	-0.001111 ***	1.030240 ***	0.019700	0.000006 ***	0.181700 ***	0.812590 ***	0.9412	2194.40	0.973
MV12	-0.000426	0.998050 ***	-0.011080	0.000005 ***	0.101890 ***	0.883840 ***	0.9287	2162.49	40.482 ***
MV13	-0.000005	0.953280 ***	-0.007140	0.000004 **	0.115550 ***	0.879550 ***	0.9322	2144.87	42.116 ***
MV14	-0.000561	0.941760 ***	-0.055210	0.000007 ***	0.141390 ***	0.842470 ***	0.9052	2136.82	102.996 ***
MV15	-0.000638	0.896930 ***	-0.009240	0.000004 **	0.126890 ***	0.875420 ***	0.8908	2063.00	53.419 ***
MV16	-0.000414	0.861660 ***	-0.038140	0.000006 ***	0.095390 ***	0.897400 ***	0.8933	1983.63	47.184 ***
MV17	0.000105	0.835260 ***	-0.037060	0.000005 ***	0.126060 ***	0.874920 ***	0.8748	1980.10	41.154 ***
MV18	-0.000019	0.796720 ***	-0.058850	0.000018 ***	0.130600 ***	0.834820 ***	0.8731	1947.60	35.406 ***
MV19	-0.000344	0.697560 ***	-0.122920 ***	0.000019 ***	0.127000 ***	0.848470 ***	0.8281	1886.99	49.962 ***
MV20	0.000251	0.658250 ***	-0.104070 ***	0.000017 ***	0.100290 ***	0.881340 ***	0.7627	1786.13	49.572 ***

Table 7 - GARCH Regressions (continued)

Varying Risk Market Model

	a_{bull}		a_{bear}^{d}		a_{bear}-a_{bull}		b_{bull}		b_{bear}^{d}		b_{bear} - b_{bull}	
MV1	-0.023059	***	-0.000169	***	0.022890	***	1.914030	***	1.171190	***	-0.742840	***
MV2	-0.016024	***	0.000460	***	0.016484	***	1.551070	***	1.176330	***	-0.374740	***
MV3	-0.011215	***	-0.001604	***	0.009611	***	1.389790	***	1.161310	***	-0.228480	***
MV4	-0.010113	***	0.000777	***	0.010890	***	1.341410	***	1.193640	***	-0.147770	***
MV5	-0.005942	***	-0.001108	***	0.004834	***	1.208540	***	1.130290	***	-0.078250	***
MV6	-0.000685		-0.000319	***	0.000366		1.108960	***	1.132480	***	0.023520	
MV7	-0.000754		0.000904	***	0.001658		1.071750	***	1.108490	***	0.036740	*
MV8	0.002590	***	-0.001181	***	-0.003771	***	0.970760	***	1.072110	***	0.101350	***
MV9	-0.000435		-0.000265	***	0.000170		1.041180	***	1.092910	***	0.051730	***
MV10	-0.001515		0.000297	***	0.001812		1.053090	***	1.088890	***	0.035800	**
MV11	0.003772	***	-0.001302		-0.005074	***	0.931420	***	1.030880	***	0.099460	***
MV12	0.003691	***	0.000012	***	-0.003679	***	0.921460	***	1.020480	***	0.099020	***
MV13	0.005075	***	0.000350	***	-0.004725	***	0.861760	***	0.988970	***	0.127210	***
MV14	0.006441	***	0.001145	***	-0.005296	***	0.799830	***	0.996840	***	0.197010	***
MV15	0.005749	***	-0.000460	***	-0.006209	***	0.793900	***	0.927680	***	0.133780	***
MV16	0.008888	***	0.000180	***	-0.008708	***	0.715940	***	0.914440	***	0.198500	***
MV17	0.008588	***	0.001411	***	-0.007177	***	0.697950	***	0.901590	***	0.203640	***
MV18	0.007877	***	-0.000365	***	-0.008242	***	0.685300	***	0.829120	***	0.143820	***
MV19	0.010023	***	0.000361	***	-0.009662	***	0.554300	***	0.768850	***	0.214550	***
MV20	0.011819	***	0.000612	***	-0.011207	***	0.495880	***	0.723160	***	0.227280	***

Table 7 (continued)

Varying Risk Market Model (continued)

	AR1	ARCH0	ARCH1	GARCH1	Total $R^{2\,a}$	LIKLHD[b]
MV1	0.004761	0.000025 ***	0.100260 ***	0.898040 ***	.8062	1269.15
MV2	0.044993	0.000013 **	0.089630 ***	0.904460 ***	.8924	1580.16
MV3	-0.025146	0.000004 **	0.078890 ***	0.920470 ***	.9152	1752.23
MV4	-0.042700	0.000010 **	0.109200 ***	0.873620 ***	.9481	1931.96
MV5	0.014743	0.000005 **	0.078060 ***	0.912730 ***	.9511	2010.34
MV6	-0.044064	0.000009 ***	0.100030 ***	0.875080 ***	.9558	2084.63
MV7	0.001262	0.000002 *	0.079080 ***	0.915010 ***	.9502	2141.55
MV8	-0.036816	0.000008 ***	0.147390 ***	0.828160 ***	.9566	2206.21
MV9	0.047610	0.000002 **	0.089600 ***	0.903640 ***	.9536	2243.64
MV10	-0.034464	0.000003 **	0.104570 ***	0.892310 ***	.9463	2184.64
MV11	0.049220	0.000006 ***	0.178350 ***	0.812220 ***	.9413	2204.04
MV12	-0.002432	0.000004 ***	0.091690 ***	0.894940 ***	.9355	2171.43
MV13	0.004592	0.000003 **	0.090820 ***	0.900820 ***	.9389	2161.93
MV14	-0.001666	0.000005 ***	0.097960 ***	0.883250 ***	.9252	2166.93
MV15	-0.013248	0.000004 **	0.118810 ***	0.878300 ***	.9041	2081.42
MV16	0.001858	0.000008 ***	0.088100 ***	0.891820 ***	.9049	2018.52
MV17	-0.004541	0.000007 ***	0.126880 ***	0.862840 ***	.8869	2013.68
MV18	-0.037669	0.000011 ***	0.097200 ***	0.878570 ***	.8837	1966.10
MV19	-0.044912	0.000012 ***	0.086520 ***	0.890250 ***	.8478	1921.54
MV20	-0.036607	0.000014 ***	0.082570 ***	0.899020 ***	.7897	1815.56

[a]The Total R^2 measures the explanatory power of both the structural model variables as well as the AR(1)-GARCH (1,1) components of the regression model.

[b]LIKLHD is the value of the log-likelihood function of the MLE regression.

[c]F-stat is an approximate f-statistic, F={(SSE1 - SSE2)/j}/{SSE2/(n-k)}, assumed to be distributed as F(j, n-k) = F(2,774), and where SSE1 and SSE2 are, respectively, the sum of squared errors from the constant risk and varying risk market models. The significance levels are approximate since the sum of squared errors are based on maximum likelihood estimation of an AR(1)-GARCH(1,1) model.

[d]The significance levels for a_{bear} and b_{bear} are approximate since they are based on comparisons of the sum of squared errors (SSE) from constrained and unconstrained regressions from the AR(1)-GARCH(1,1) models using maximum likelihood estimation.