Modeling Returns Generating Processes and Tests of the APT

Published in the *2000 Proceedings of the Academy of Economics and Finance*

J. Edward Graham and Robert T. Burrus, Jr., University of North Carolina at Wilmington

Abstract

Two approaches to the multi-factor modeling of security returns are evaluated. In the first, a variance/covariance matrix of security returns and macroeconomic factors is assembled and decomposed. Factors are generated through this algebraic decomposition and are thus unspecified. In the second, a macroeconomic variable model is proposed and a linear model estimating security returns as a function of innovations in the macroeconomic variables is developed. The factors are defined as the innovations in the macroeconomic variables. Tests of the Arbitrage Pricing Theory (APT) using the factor analytic and macroeconomic variable models are examined. An evaluation of the ability of the APT to model expected returns is conducted. Given certain disadvantages of the factor analytic approach and the intuitive appeal of the macroeconomic variable model, this second approach is favored.

Introduction

Assuming that asset prices depend on their exposure to state variables in the economy, how well does factor analysis illustrate this exposure? How well do parameter estimates for innovations in pre-specified macroeconomic variables describe security returns? What are the implications of the answers to these questions for tests of the Arbitrage Pricing Theory (APT)? These three questions are addressed in the following pages.

In the next section, the multi-factor modeling of security returns and a review of the APT is provided. Implementation and testing of the APT through factor analysis is covered in Section III. Applications of the APT through the modeling of macroeconomic innovations are considered in Section IV. A summary and encouragement for subsequent research closes the paper.[1]

Background of the APT

An underlying premise of the APT is that security prices respond to a shared set of systematic or Astate@ factors. Supporting this premise, there exist industrial, utility and transportation indexes implying that investors think of the included stocks as responding to common factors. Examinations by King (1966) and by Elton and Gruber (1973) respond to this implication. King applies factor analysis to the covariance matrix of a Alarge set@ of serial stock price changes and discovers a set of pervasive market and industry factors. Elton and Gruber consider an Aalternative behavioral model@ that assumes stocks respond to common factors.[2]

Long (1974) describes expected returns with three factors: the term structure of interest rates, inflation expectations and the spread between riskless and risky security returns. Fama and MacBeth (1973) likewise affirm the appropriateness of a multi-factor model if the one-factor return generating process has correlated residuals. The macroeconomic factors modeled by Long and the multiple risk premiums selected by Fama and MacBeth share the intuition of the APT[3]: that it is desirable to view economic factors simultaneously in the context of a single model.

In proposing the APT, Ross (1976) holds that systematic variability alone affects expected returns. The APT differs in two distinct ways from the Sharpe, Lintner, Black Capital Asset Pricing Model (CAPM); no particular Amarket portfolio@ plays a role in the APT and the APT allows for more than one generating factor. The theory begins with the traditional assumptions of perfectly competitive and frictionless markets, with homogeneous beliefs among investors that expected returns are governed by a K-factor linear return generating model of the form:

$$(1) \quad r_i = E_i + b_{i1}d_1 + ... + b_{ik}d_k + e_{i,} \; i = 1 ,...., n$$

where the return on asset i is a function of asset i=s expected return and the sensitivity (b) of the asset to each common factor (d). Roll and Ross (1980) propose that estimated expected returns depend on estimated factor loadings; and variables such as Ae_i@ above (the random disturbances or noise) do not add any explanatory value to the model. If they did, there would be a need for more than the k hypothesized factors.

The ability of the APT to illustrate the exposure of asset-prices to state variables can be considered in two ways. In the first, factor coefficients and expected returns are estimated from serial tests on individual asset returns; these estimates are then used to test the cross-sectional pricing conclusions of the APT. The b coefficients are estimated through factor analysis (a decomposition of the covariance matrix of the assets being examined). The APT=s premise is that excess expected returns are accounted for, on average, by these factor loadings.

The second approach, originally investigated by Chen, et al [1983, 1986], limits the use of factor analysis. Instead, the sensitivity of returns to Ainnovations@ in a group of macroeconomic variables is measured. Pre-specified macroeconomic variables are used to describe returns. As with the factor analysis in the first approach, the APT proposes that the discrepancies in equation (1) are a linear function, on average, of these economic factors.

Tests of the APT Through Factor Analysis

Roll and Ross (RR) (1980) conduct a three-part examination of the ability of the Apt to model expected returns through factor analysis. They first test the APT=s ability to model returns. They then examine the correlations between residuals and, finally, they consider the difference between factor structures across groups of securities; RR examine 42 groups of 30 alphabetically sorted securities listed on the New York and American Stock Exchanges between 1962 and 1972. Although their data seem to support the APT, they remind their readers of the preliminary and admittedly Aweak@ nature of their tests. Lehman and Modest (1988) note that these tests often lack the power to reject the theory when it is false.

Shanken (1982, 1985) responds to RR=s work. He allows that there are two primary problems with the decomposition of the variance/covariance matrix of returns. First, the number of factors needed to complete the model is indeterminate and second, the factors themselves may not be unique. He shows that the identification of factors is sample specific B that determining which factors are priced is difficult. He posits that the solution provided through factor analysis is not unique. Shanken=s arguments concerning the number of factors are echoed in Dhrymes et al (1984), Brown (1989) and Trzcinka (1986). Trzcinka acknowledges there is Ano obvious way to choose the number of factors,@ yet he is attracted to the Asimpler structure [than CAPM] and richer implications@ of the APT. He, Brown and Dhrymes et al, find that as the number of securities in a sample increases, the number of derived factors also increases. Connor and Korajczyk (1993) consider the number of factors, as well.

A central theorum of the APT is that the mean-squared-error terms in equation (1) converge to 0. Shanken posits that, given the need for an infinite number of assets, it can=t be shown that these terms converge at all. He proposes a theoretical argument against the testability of the APT, through factor analysis, similar to the Auntestability@ of the CAPM suggested by Roll (1977). Further, he shows that if the factors from even a single sample are rotated, new eigenvectors and eigenvalues are generated.[4] As an extension, it becomes impossible to infer which specific economic factors underlay factor analysis. (See Elton and Gruber 1973).

Two critical hypotheses are implied by the APT B one that the intercept term equals the risk-free rate and a second that there is a linear relationship between the risk measures and expected returns. Gultekin and Gultekin

(1987) posit that these two hypotheses are sensitive not only to the issues mentioned above, but also to such anomalies as the January effect and firm size. Rejection of the APT, based on its statistical sensitivity to such factors as firm size, is discussed also by Reinganum (1981) and Robin and Shukla (1991).

The ability of a measure of unsystematic risk (such as firm size) to explain risk-adjusted returns violates the theory of the APT. Although Lehman and Modest (1988) suggest that the APT is pricing Amost listed securities with little error,@ they nonetheless acknowledge this deficiency. They likewise recognize the inherent problems encountered in measuring common factors implicitly through factor analysis. Brown (1989) concludes that a purely statistical technique may lead to false conclusions.

Chen (1983) notes that the development of the theory of arbitrage pricing is Aquite separate from the factor analysis.@ Factor analysis is used only as a tool to uncover the pervasive forces in the economy by examining how asset returns covary together. He admits that factor analysis can produce many different factor structures from the manipulated portfolios. Although he supports the APT in testing it against the explanatory power of firm size, he concludes that the APT is designed Amore in the spirit@ of macroeconomic variable modeling. The question then becomes: If the APT can=t be adequately tested by statistical factor analysis, how might it be tested?

The APT and Macroeconomic Variable Modeling

In the macroeconomic variable model, the factors are defined as Ainnovations in a set of macrovariables.@ In contrast to the factor loading model, that generates factors algebraically, the macroeconomic variable model uses prespecified economic factors to model expected returns. Chen, Roll and Ross (1986) (CRR) affirm that factors are orthogonal in linear factor analysis B the factors are independent. This seems to defy common sense. One expects changes in such modeled variables as inflation expectations and the term structure of interest rates to have significant cross-sectional dependence. This is intuitive. However, the marginal changes in these factors (or innovations) are orthogonal (independent). King (1966) notes that instantaneous discounting of news arrivals brings about temporal independence of successive changes in a state variable class. By measuring these instantaneous changes or innovations, macroeconomic variables lend themselves to modeling within the APT.

Roll and Ross (1984) claim that an asset=s riskiness is the sensitivity of its return to Aunanticipated changes in four economic variables B inflation, industrial production, risk premia, and the slope of the term structure of interest rates.@ Different assets are presupposed to have different sensitivities to these systematic factors.

Recognizing that the APT is silent about which economic events influence returns, CRR (1986) explore the identification of these state variables. The major thrust of their effort is to examine the relation between non-equity economic variables and stock returns. CRR specify seven pervasive or systematic influences as likely sources of investment risk. These include innovations in industrial production, inflation expectations, risk premia, the term structure of interest rates, selected market indices, changes in real consumption and oil prices. A sample of assets is chosen and the assets= exposure to the economic state variables is estimated. CRR estimate a linear model using asset returns and the innovations of the variables mentioned above. A cross-sectional regression provides an estimate of the risk premium associated with a given state variable. They then test the significance of the relationships.

Using the state variables outlined above, CRR assume that Aindividual stock returns follow a factor model of the form@:

$$(2) \quad R = á + â_1\, MP + â_2\, DE1 + â_3\, UI + â_4\, UPR + â_5\, UTS + e,$$

where á is the constant term, the betas are the loadings on the state variables and e is an idiosyncratic error term. The independent variables employed by CRR include:

(1) MP - monthly growth rate in industrial production.
(2) DE1 - change in expected inflation.
(3) UI - unanticipated inflation.
(4) UPR - unanticipated changes in risk premiums.
(5) UTS - unanticipated changes in the term structure of interest rates.

The authors reach conclusions consistent with the APT. As the model above indicates, they discover significant explanatory value in several of the variables. Industrial production, changes in the risk premiums and Atwists in the yield curve@ are found to be significant in explaining returns. Somewhat more weakly, the innovations in the two inflation measures are also important in explaining stock returns. They find these returns are Aexposed to systematic economic news, that they are priced in accordance with their exposures and that the news can be measured as innovations in state variables.@They conclude that simple and intuitive financial theory (i.e. the APT) can illustrate this exposure satisfactorily.

The technical work of CRR indicates the types of risk modeled in equation (2) are significant for determining realized and expected returns. In other words, the five different types of risk have non-zero APT prices; Athey are relevant risks, influencing equilibrium returns.@ Surprisingly, CRR find that the value-weighted NYSE index, that explains a significant position of the time series variability of stock returns, has an insignificant influence on pricing when compared against the economic state variables. Likewise, the consumption and oil-price variables are not found to be significant.

Equation (1) is a returns generating process where returns differ from expectations by a discrepancy that is a linear function of the factors being modeled. This discrepancy is unanticipated. Using factor analysis, Brown & Weinstein (1983) confirm that the number of pervasive factors is probably no greater than five. Although the factor analytic technique they use does not lend itself to economic interpretation, it still follows that a similar number of macroeconomic variables are at play; a similar number of economic factors explain the deviations of returns from expectations. They then measure four such factors, identified below, using four macroeconomic variables.[5] The innovations of these factors are approximately mean zero and uncorrelated with each other or the residuals, as the model requires. Brown and Weinstein find, using ordinary least squares time-series regressions, that the macroeconomic measures Ahave influences as predicted and that they explain a credible variation in stock market returns;@ all four are statistically significant. They conduct their serial tests on three alternative portfolios: the S&P 500 index, an equally weighted portfolio of 20 randomly selected stocks and the T. Rowe Price New Horizons Fund. The authors estimate factor sensitivities twice: once using a constant risk-free rate and once using a non-constant rate. The risk free rate is a pivotal element of the linear process in the APT. Their tests for differences in the factor coefficients between constant and non-constant risk free rates adds support to their conclusions. Brown and Weinstein (1983) note that in factor analysis, the risk-free rate should be constant across securities.

When Brown and Weinstein add a fifth factor B the Aunanticipated change...not explained by the first macroeconomic factors@ B they generate significant results. Their model, adapted to some of the expressions used in equations (1) and (2), if of the form:

$$(3) \quad P_i(t) = E_i + \hat{a}_{i1}\,UPR(t) + \hat{a}_{i2}\,UTS(t) + \hat{a}_{i3}\,UI(t) + \hat{a}_{i4}\,UGS(t) + \hat{a}_{i5}\,UM(t) + e_i(t),$$

where the returns on asset i at the end of period t are a function of asset i=s expected return, its sensitivities in period t to the five selected macroeconomic variables and an error term. The selected variables in the B & W model:

(1) UPR(t) - Unexpected change in risk premiums or default risk measured as the return on corporate bonds in period t minus the return on government bonds in period t.

(2) UTS(t) - Unexpected change in the term structure measured as the return on government bonds in period t minus the return on treasury bills in period t.

(3) UI(t) - Unexpected inflation measured as a function of unobserved expected inflation and the implicit deflator for non-durable consumption goods.

(4) UGS(t) - Unanticipated change in the growth rate of real final sales.

(5) UM(t) - Unanticipated change in the market index not explained by the first macroeconomic factors (the estimated residuals from the OLS regression.)

The authors find that Athe measured macroeconomic factors appear to work for individual security returns [and] have different effects on different assets as predicted by the APT.@ They identify factor measures to explain actual and expected returns consistent with the retum generating mechanism and no arbitrage condition of the APT. They posit that the APT illustrates asset returns well using macroeconomics variable modeling.

Berry, Burmeister and McElroy (1988) (BBM) show that an APT model that incorporates five selected macroeconomic variables is superior to CAPM. It does Anot depend on any particular market index.@ They estimate the sensitivities (factor loadings) of securities to known APT factors. BBM use an APT model that incorporates unanticipated changes in the five macroeconomic variables of Burmeister and Wall (1986) and Chen, Roll and Ross (1986)- default risk, the term structure of interest rates, inflation or deflation, residual market risk and the long-run expected growth rate of profits for the economy. A consensus in the literature begins to develop with the publication of the work of BBM.

One assumption of the APT is that risk factors can be used to model expected returns. According to BBM, these risk factors must be Aunpredictable to the market, have a pervasive influence on stock returns and must influence expected return.@ Otherwise, the pricing of these factors though the APT becomes problematic. In order for the orthogonal or independent factors to nest within the linear construction of the APT, they must be unpredictable (independent). And to be priced, these factors must have a pervasive influence on returns.[6]

BBM use APT risk profiles to measure the impact of these factors in selected market sectors and industries. Earlier work by Burmeister and Wall (1986) finds that different factors influence different assets to varying degrees. BBM extend this to an examination of the sensitivities of specified assets (or groups of assets) to each of the five factors. They show that the APT models expected returns well, but are able to highlight a few of its deficiencies. An APT strategy is, for example, handicapped where an accurate forecast of factor realizations (of macroeconomic Anews@) is predicted. Elton and Gruber (1973) and Long (1974) also mention the evasive nature of manageable macroeconomic data. At the heart of accurate modeling of expected returns by the APT (using macroeconomic variables), is the timeliness and Acleanliness@ of the macroeconomic data. Another issue to which they allude are the changing intertemporal sensitivities of given assets to each of the factors. These sensitivities or factor loadings do change and need to be regularly reevaluated. BBM=s objective is to use the APT to determine sensitivities of given assets to each of the factors. The model is then used to assemble portfolios with pre-determined risk exposures. They observe that different stocks offer very different profiles of risk exposure. Inflation and the term structure of interest rates, for example, have different impacts in different industries. BBM note also that unexpected inflation has only a minor effect on the returns of industries such as utilities where passing on inflationary costs in the form of higher prices is relatively easy.

Chen and Jordan (1993) examine the APT using both the factor loading and macroeconomic variable models. In deriving their factor loading model, they use the matrix decomposition considered above. They use the macroeconomic innovations of CRR (1986) in their second approach. They find no significant differance in test results between the two models. Nonetheless, given certain desirable features of the macroeconomic model, Chen and Jordan are able to suggest its desirability in all three of their tests. They acknowledge that there is a lack of a formal discipline in the selection of the macroeconomic factors to be modeled. They still find that the APT models expected returns well using macroeconomic innovations. It provides Aan effective means for managing different types of risk to which investors are exposed.@ Greater computer power and a reliable database of asset returns and risk factor realizations favor the implementation of APT-based strategies.

Summary and Conclusions

The APT models returns as a linear function of an asset=s exposure to pervasive state variables. These state variables are the factors that influence returns and that are hypothesized to be priced by the APT. An underlying assumption of the APT is the no-arbitrage condition; this provides that the factors are priced and that deviations of an asset=s returns from expectations can be Aexplained.@ The identification and modeling of the asset=s exposure to these sundry factors provides this explanation.

Two approaches used to identify and model these factors have been considered. The first approach provides for the construction and decomposition of a variance/covariance matrix of returns. This allows for the identifications of factors that are statistical constructs of some underlying source of risk. The second approach provides for the cross-sectional regression of pre-specified macroeconomic factors against security returns.

This paper highlights some of the disadvantages of a factor analytic approach to the APT. This approach is subject to Astatistical problems peculiar to the use of factor analysis.@ Shanken (1982) and Dhrymes et al (1984) note the sensitivity of factor analysis to the size and nature of the sample under study. Shanken also brings into question the entire concept of the factor analytic approach to the APT.

The macroeconomic variable modeling approach to the APT has two distinct advantages. First, it is not subject to the statistical peculiarities of the factor analytic approach. Second, given the pre-specification of the APT factors (the macroeconomic source of security price/return variation), this approach provides a direct link between various financial strategies and changing economic events. The primary disadvantage of this macroeconomic approach is the lack of clear and accepted guidance for choosing the variables.

With particular emphasis on the comments of Shanken (1982,1985), comprehensive challenges to the factor analytic approach are recognized. The initial work of Chen (1983), and later work by Chen, et al (1986) and Berry et al (1988) illustrate the ability of the APT to model expected returns using pre-specified macroeconomic variables.

Given the economically interpretable factors of the macroeconomic model, it has intuitive appeal. Likewise, the unresolved theoretical challenges to the factor loading model do not encourage its empirical application. The APT is shown to be viable, on average, in illustrating the exposure of security returns to state variables in the economy. Subsequent examinations of the macroeconomic model using a different set of macrovariables or improved measures of the variables used in prior studies are encouraged. As observed by Long (1974), Athe discovery of previously unperceived empirical regularities provides an a priori basis for further advancement of theory.@

Notes

1. Originally proposed by Ross (1976), the APT is later supported by Roll and Ross (1980) and Chen (1983). The work of Roll and Ross is countered, in a lively exchange, by Shanken (1982, 1985) and Dhrymes, et. al (1984). Work by Eun and Resnick (1992)and Connor and Korajczyk (1993) continue the debate .

2. The literature is replete with attempts to forecast security prices based upon sundry multi-factor modelings. Recent studies by Fama and French (1989, 1993) provide a review of a number of salient studies.

3. Both the work of Berry et. al (1988) and Chen et. al (1986) model the term structure of interest rates and inflation expectations, in macroeconomic applications of the APT.

4. An analysis of linear algebra and the generation of eigenvalues and eigenvectors through matrix decomposition is provided in Chiang (1984), pp. 54-124.

5. Burmeister and Wall (1986) study the linkage between stock market returns and macroeconomic activity. They note the clouded mathematics of factor analysis and focus on macroeconomic measures of the APT factors. The articles by Chen, Roll and Ross (1986) and by Burmeister and Wall (1986) contain the details of the techniques the authors used in their respective studies. These contain specific identifications of the variables being modeled. Since the purpose here is to consider how well the APT models security returns given macroeconomic exposure, the details of their modeling (in the interest of brevity) are omitted.

6. Recall the nature of the oil price and consumption-based factors of Chen, Roll & Ross (1986). The innovations of these factors, while pervasive, are not found to have a significant influence on security returns.

References

Berry, Michael, Edwin Burmeister and Marjorie McElroy. 1988. ASorting Out Risks Using Known APT Factors.@

Financial Analysts Journal 44: 29-42.

Brown, Stephen. 1989. AThe Number of Factors in Security Returns.@ *Journal of Finance* 44: 1247-1262.

Brown, Stephen and Mark Weinstein. 1983. AA New Approach to Testing Asset Pricing Models : The Bilinear Paradigm.@ *Journal of Finance* 38: 711-744.

Burmeister, Edwin and Kent Wall. 1986. AThe Arbitrage Pricing Theory and Macroeconomic Factor Measures.@ *The Financial Review* 21: 1-20.

Chen, Nai-Fu. 1983. ASome Empirical Tests of the Theory of Arbitrage Pricing.@ *Journal of Finance* 38: 1393-1414.

Chen, Nai-Fu, Richard Roll & Stephen Ross. 1986. AEconomic Forces and the Stock Market.@ *Journal of Business* 59: 383-403.

Chen, Su-Jane and Bradford Jordan. 1993. ASome Empirical Tests in the Arbitrage Pricing Theory: Macrovariables vs. Derived Factors.@ *Journal of Banking and Finance* 17: 65-89.

Chiang ,Alpha C. 1984. *Fundamental Methods of Mathematical Economics*, Third Edition. New York: McGraw-Hill.

Connor, Gregory and Robery Korajczyk. 1993. AA Test for the Number of Factors in an Approximate Factor Model.@ *Journal of Finance* 48: 1263-1292.

Dhrymes, Phoebus, Irwin Friend and N. Bulent Gultekin. 1984. AA Critical Re-examination of the Empirical Evidence on the Arbitrage Pricing Theory.@ *Journal of Finance* 39: 323-346.

Elton, Edwin and Martin Gruber. 1973. AEstimating the Dependence Structure of Share Prices B Implications for Portfolio Selection.@ *Journal of Finance* 28: 1203-1232.

Eun, Cheol and Bruce Resnick. 1992. AForecasting the Correlation Structure of Share Prices: A Test of New Models.@ *Journal of Banking and Finance* 16: 643-656.

Fama, Eugene and Kenneth French. 1989. ABusiness Conditions and Expected Returns on Stocks and Bonds.@ *Journal of Financial Economics* 25: 23-49.

Fama, Eugene and Kenneth French. 1993. ACommon Risk Factors in the Returns on Stocks and Bonds.@ *Journal of Financial Economics* 33: 3-56.

Fama, Eugene and James MacBeth. 1973. ARisk, Return and Equilibrium: Empirical Tests.@ *Journal of Political Economy* 81: 607-636.

Gultekin, Mustafa and Bulent Gultekin. 1987. AStock Return Anomalies and Tests of the APT.@ *Journal of Finance* 42: 1213-1224.

King, Benjamen F. 1966. AMarket and Industry Factors in Stock Price Behavior.@ *Journal of Business* 39: 139-191.

Lehman, Bruce and David Modest. 1988. AThe Empirical Foundations of the Arbitrage Pricing Theory.@ *Journal of Financial Economics* 21: 213-254.

Long, John 1974. AStock Prices, Inflation and the Term Structure of Interest Rates.@ *Journal of Financial Economics* 1: 131-170.

Reinganum, Marc. 1981. AEmpirical Tests of Multi-factor Pricing Model: The APT, Some Empirical Results.@ *Journal of Finance* 36: 313-321.

Robin, Ashok and Raui Shukla. 1991. AThe Magnitude of Pricing Errors in the Arbitrage Pricing Theory.@ *Journal of Financial Research* 14: 65-82.

Roll, Richard. 1977. AA Critique of the Asset Pricing Theory=s Tests.@ *Journal of Financial Economics* 4: 129-176.

Roll, Richard and Stephen Ross. 1980. AAn Empirical Investigation of the Arbitrage Pricing Theory.@ *Journal of Finance* 35: 1073-1103.

Roll, Richard and Stephen Ross. 1984. AThe Arbitrage Pricing Theory Approach to Strategic Portfolio Planning.@ *Financial Analysts Journal* 40: 4-26.

Ross, Stephen . 1976. AThe Arbitrage Theory of Capital Asset Pricing.@ *Journal of Economic Theory* 13: 341-360.

Shanken, Jay. 1982. AThe Arbitrage Pricing Theory: Is It Testable?@ *Journal of Finance* 37: 1129-1140.

Shanken, Jay. 1985. AMulti-Beta CAPM or Equilibrium APT: A Reply.@ *Journal of Finance* 40: 1189-1196.

Sharpe, William. 1964. ACapital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk.@ *Journal of Finance* 19: 425-442.

Trzcinka, Charles. 1986. AOn the Number of Factors in the Arbitrage Pricing Model.@ *Journal of Finance* 41: 347-368.