Econ 325 Handout on Optimal Extraction and the “r-percent” rule
Keep in mind that deciding how quickly to extract the nonrenewable natural resource is a type of investment problem. If you are the landowner then your mine is an asset. The value of your mine is the value of your mineral asset left in the ground. That is, holding onto this asset means not extracting the resource. When you extract, you are giving up a portion of the asset and “cashing it in” by selling it. As is the case with all investments, which asset you purchase and how long you hold onto it, depend on the expected rate of return from that asset – how much its value will increase over time. Obviously, you want to purchase the asset with the highest return. However, in a perfectly competitive market with no uncertainty, all assets must have the same rate of return in equilibrium.

For example, consider an economy that has only two equally risky assets (A and B). Currently asset A increases in value 10 percent per year, and asset B increases in value 20 percent per year. Clearly, no one will want to invest in asset A and everyone will want to invest in asset B. The price of the high return asset will therefore increase, so its return (what you get back after price, or the “rent”) will decrease, while the price of the low return asset will decrease. Eventually, their returns will be equal.

The r-percent rule says that to maximize profits from the extraction of a mineral resource over a period of time, you should extract an amount each period such that the marginal profit from the mine (the per unit price you will receive less marginal extraction costs) increases at a rate equal to the rate of interest. That is, you want to extract whatever amount it takes each period so that the value of your mine asset is growing at the same rate as other assets.

If you extract at such a rate that the value of your mine is growing faster than other assets then you are extracting “too fast”. You’re taking value out of the resource asset when it’s doing well. In other words, you are better off extracting slower, and leaving that value in the mine (rather than some other asset), where it earns a better rate of return. But, doing so results in less scarcity, and a lower rate of return.

If you extract at such a rate that the value of your mine is growing slower than other assets, then you are extracting “too slow”. You’re leaving value in the slower growing asset, you should “cash in” and put the value in another asset. In other words, you would have been better off extracting faster, and taking the value out of the mine to be invested in another asset where it earns a better rate of return. But, extracting faster results in more scarcity and a higher rate of return. So, either of the above deviations from the optimal rate of extraction is not profit maximizing. As you alter your extraction rate to correct for this, you set up the conditions for satisfying the r-percent rule.

Our proof of the r-percent rule of optimal extraction relies on an application of the equimarginal principle over time. That is, to maximize profits over a given time period, you must have the marginal profit equal in all periods. This is because we assume that, due to diminishing marginal returns, the marginal returns from capital assets are decreasing in the quantity invested in the asset. See illustration below.

Interest rate (percent)

Marginal return = demand for interest-bearing asset

Quantity of capital
We can assume that the same principle applies to the quantity of mineral resource in our mine. The higher the quantity, the lower the return, because the resource is not as scarce (and vice versa).

Also keep in mind that we are assuming that marginal costs of extraction (both physical and opportunity costs) are increasing in the amount extracted.

<table>
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<tr>
<th>Costs ($)</th>
<th>Marginal Cost of extraction</th>
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<td>Quantity extracted</td>
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Now to illustrate the application of the equimarginal principle, consider two periods (t and t+1). If the return from your mine in period t+1 is greater than that in period t, you are better off holding back extraction till period t+1. In doing so, you decrease the marginal extraction costs in period t (by decreasing the quantity extracted), hence earn a higher marginal return given a constant price. Similarly when you extract more in period t+1 you increase marginal extraction costs and therefore earn a lower return.

As long as the returns are different, you can increase profits by reallocating extraction towards the higher return period. By reallocating extraction toward the higher return period to maximize profits, the returns will equalize. Note that this works much like the two-asset example above.

Summary:
- Return from the mine = present value of marginal profit = price less extraction costs.
- Profits are maximized when the return from the mine is equal across periods.
- Equality of marginal profits across periods requires that marginal profit increase at a rate exactly equal to the market rate of return (the interest rate).
- This means that to maximize profits, the landowner will extract an amount each period such that the marginal profit from the mine increases at a rate equal to the rate of interest.