Consider an economy for which the Efficient Market Hypothesis holds and in which all financial assets are possibly traded (abusing words we call this The Complete Markets assumption). Then the two proposition of Miller and Modigliani are:

- **MM1** - The value of the firm is independent of its capital structure (the proportion of debt and equity used to finance the firm’s operations).

- **MM2** - Provided the firm’s investments (projects) are unchanged, its value of the firm is independent of the firm’s dividend policy.

The value of a firm \( V \) is defined to be the sum of the value of the firm’s debt and equity. We usually use \( B \) or \( D \) for the value of debt and \( S \) or \( E \) for the value of equity.

\[
V = B + S = D + E
\]

Shareholders are interested in maximizing the value of the firm’s shares. A key question is: What is the ratio of debt to equity, if any, that maximizes the shareholder’s value?

As it turns out, changes in capital structure benefit shareholders if and only if the value of the firm increases. So we concern ourselves with the question of which proportion of debt versus equity maximizes the overall value of the firm.

We assume perfect capital markets in the following analysis:
- Perfect competition
- Equal access to information
- No transaction costs
- No taxes (later we drop this assumption)

We also assume that the individual can borrow at the same rate as the firm.

Example:
We will compare a firm whose capital structure is all equity, with a firm that is identical, except that it has some debt in its capital structure. Suppose a firm has 1000 shares, a share price of $10, a market value of $10,000, and operating income as described in the table below. This income is perpetuity, and all earnings are paid out as a dividend to investors. Expected income is $1500 and expected return is 15%.

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating income</td>
<td>500</td>
<td>1000</td>
<td>1500</td>
<td>2000</td>
</tr>
<tr>
<td>Earnings Per Share</td>
<td>.5</td>
<td>1.00</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Return on Equity</td>
<td>5%</td>
<td>10%</td>
<td>15%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Suppose the firm borrows $5,000 at 10% rate of interest and buys back 500 of its shares to create the following new situation: # shares 500, \( B = 5000 \), \( S = 5000 \), shares value: $10, annual interest: $500:

<table>
<thead>
<tr>
<th>Operating Income</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>
Argument that value has increased: If operating income is greater than $1,000 then the expected Return On Equity increases. Since we expect operating income of $1,500, leverage must have added value. Shareholders expect 20% ROE with leverage. Counter-argument: Suppose an investor takes $10 of his/her own money and borrows $10 at 10% interest and invests the $20 to buy 2 shares of the unlevered firm.

Note: Investor gets the same result from borrowing personally as he/she gets from investing in the leveraged firm. Therefore, the firm is not doing anything by leveraging that the investor could not do on his/her own. This implies $V_L = V_U$. This is the famous Modigliani-Miller Proposition I.

So leverage increases the expected earnings per share, but not the share price. How can that be? Leverage also increases the riskiness of the earnings. We must discount those higher expected earnings at a higher discount rate.

All equity: $1.50/0.15 = $10$
Leveraged: $2.00/0.20 = $10$

Recall: The value of a firm = the PV of the future cash flows.

If the firm is all equity financed, then $V_U = C_1/(1+r_0) + C_2/(1+r_0)^2 + \ldots$ where $r_0$ is the cost of capital of an all equity financed firm and $C_K$ is the Cash Flow for period $K$. If there is some debt in the capital structure, the cost of capital is given by:

$$r = \frac{B}{(B+S)}r_B + \frac{S}{(B+S)}r_S$$

where $r_S$ is the cost of equity capital of the now leveraged firm and

$$V_L = C_1/(1+r) + C_2/(1+r)^2 + \ldots$$

Since the value of the company doesn’t change with leverage, $V_L = V_U$, and so $r$ must equal $r_0$; that is, $r$ must be constant as debt is added to the capital structure (remember here we are assuming no taxes etc.). The idea is that as “expensive” equity is replaced with “cheap” debt, the risk to equity increases, as a result of the increased leverage, and so $r_S$ increases. The increase in $r_S$ just balances against the increased fraction of cheaper debt, so that $r$ stays constant.

$r_0 = \text{cost of capital for the all equity firm} = r_a$. This is the required/expected return on the assets – reflects business risk, not financial leverage. If we solve:

$$r_0 = \frac{B}{(B+S)}r_B + \frac{S}{(B+S)}r_S \text{ for } r_S, \text{ we get: } r_S = r_0 + \frac{B}{S}(r_0 - r_B)$$

This is Modigliani-Miller Proposition II.

In our example above, $r_a = 1500/10,000 = 15\%$.
If $B = S = 5000$, then:

$r_S = 15\% + \frac{5000}{5000}(15\% - 10\%) = 20\%$
Initially the firm was all equity, had 1,000 shares at $10/share and equity worth $10,000:

\[ V = \frac{1500}{.15} = \frac{(CF/r_s)}{10,000} \]

With $5,000 debt at 10% and $5,000 equity, now \( r_s = 20\% \), up from 15% and:

\[ V = \frac{500}{.10} + \frac{1000}{.20} = \frac{interest\ payment}{r_B} + \frac{CF}{r_s} = 5,000 + 5,000 = 10,000 \]

Essentially, we split a single cash flow of 1500/year, discounted at 15% into 2 cash flows of 500 and 1000, the former discounted at 10% and the latter discounted at 20%.

Let’s redo this example, assuming the 1500 expected cash flow was pre tax and taxes at 35%:

<table>
<thead>
<tr>
<th></th>
<th>Unleveraged</th>
<th>Leveraged</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT (expected)</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>Interest</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>Pretax income</td>
<td>1500</td>
<td>1000</td>
</tr>
<tr>
<td>Taxes</td>
<td>525</td>
<td>350</td>
</tr>
<tr>
<td>Net Income (to Shareholders)</td>
<td>975</td>
<td>650</td>
</tr>
<tr>
<td>Total Income to debt and equity investors</td>
<td>975</td>
<td>1150</td>
</tr>
<tr>
<td>Interest Tax Shield</td>
<td></td>
<td>175</td>
</tr>
</tbody>
</table>

So, case I: UN-Levered:

\[ CF = 1500, \]
\[ 525 \text{ are taxes paid,} \]
\[ 975 \text{ to equity.} \]

and \[ V_U = \frac{975}{.15} = 6,500 \]

Case II: Levered:

\[ CF = 1500, \]
\[ Interest: 500, \]
\[ Taxes: 350, \]
\[ 650 \text{ to equity.} \]

and \[ V_L = \frac{500}{.10} + \frac{650}{.20} = \frac{5,000 + 3,250}{1,750} = 6,500 + 1,750 = 8,250 \]

Note that in Case II, more cash flows to the “good guys” – the investors in the firm (both creditors and shareholders), and less cash flows to the “bad guys” – the government, via taxes.

Note that the value of the firm has increased by $1,750; which is $175/.10 – the present value of the annual cash flow shielded from taxes – that is, PV(ITS). (How do we justify discounting at 10%?)

Note that: \[ V_L = 6,500 + 1,750 = V_U + PV(ITS) \]

In general, the annual tax shield is \( r_B BT_c \), and the PV of the tax shield is \( (r_B BT_c)/r_B = BT_c \).
So, we can conclude, in the presence of taxes that:  \( V_L = V_U + PV(\text{Interest Tax Shield}) \). If the debt is fixed and permanent, \( V_L = V_U + BT_c \).

Brief digression:

In fact, if we also include the expected cost of financial distress as a result of the increased debt, we get:

\[ V_L = V_U + PV(\text{ITS}) - PV(\text{costs of financial distress}) \]

This analysis leads to a theory called the **trade off theory** of capital structure. The optimal capital structure reflects a tradeoff between the tax benefits of debt due to the deductibility of interest expense, versus the increased risks of financial distress due to the high(er) debt levels.

Anyway, let’s continue with our example:

**MM Proposition II** – in the presence of taxes, can be shown to be:

\[ r_s = r_0 + \frac{B}{S}(1-T_c)(r_0 - r_B) \]

We are still assuming \( B = \$5,000 \) at 10% interest, and since \( V_L = \$8,250 \), therefore \( S = \$3,250 \).

In the presence of taxes and leverage, \( r_s = 0.15 + \frac{5000}{3250}(1-0.35)(0.15-0.10) = 0.20 \)

To verify:  
\[
S = \frac{\text{EBIT} - r_BB(1-T_c)}{r_s} = \frac{1500 - 500(1-0.35)}{0.20} = 3250
\]

What is the after tax, weighted average cost of capital – WACC?

\[
\text{WACC} = \frac{B}{V_L}r_B(1-T_c) + \frac{S}{V_L}r_s = \frac{5000}{8250}(0.10)(1-0.35) + \frac{3250}{8250}(0.20) = 0.1182
\]

So the cost of capital reduces from 15% with no debt to 11.82% with debt, and it is NOT because debt is cheaper than equity, but rather because of the tax deductibility of debt!

Note that \( V_L = \text{EBIT}(1-T_c)/r_{\text{WACC}} = 1500(1-0.35)/0.1182 = 8250 \) as before. It is standard practice to discount the after tax operating cash flows of the unleveraged firm by the **WACC**. The tax benefit of debt is embedded in the WACC formula.

Let’s think about this some more….

Suppose the firm was initially all equity, and moved to \( \$5,000 \) of debt.

The all equity firm was worth \( \$6,500 = 1500(1-0.35)/0.15 \)

Suppose there were 1,000 shares worth \( \$6.50/s\)hare.

When the firm announced that it would borrow \( \$5,000 \) at 10% interest and use the debt to buy back some of its shares (that is, its equity), the value of the firm increased immediately (in efficient capital markets) by \( \$1750 = PV(\text{ITS}) \).
Since there were still 1,000 shares outstanding, the shares increased in value by $1.75, so the share price jumped to $8.25.

Shortly thereafter, the $5,000 was actually borrowed. How many shares were bought back?

$5,000/8.25 = 606 shares, leaving 394 shares remaining.

The 394 shares are worth 394 x $8.25 = $3250 and B = $5,000

Note: Shareholders holding shares at the time of the announcement of the leveraged recapitalization capture the benefit – the tax benefit of debt, whether or not they choose to tender their shares.